1. Appendix A: Definition of Stationary Equilibrium

In the benchmark economy, we restrict ourselves to stationary equilibria. The individual state variables are deposit holdings, d, mortgage balances, m, housing stock holdings, h, and the household wage, w; with $x = (w, d, m, h)$ denoting the individual state vector. Let $d \in \mathcal{D} = \mathbb{R}_+$, $m \in \mathcal{M} = \mathbb{R}_+$, $h \in \mathcal{H} = \{0, h_1, ..., h_{11}\}$, and $w \in \mathcal{W} = \{w_1, ..., w_7\}$, and let $\mathcal{S} = \mathcal{D}\times\mathcal{M}\times\mathcal{H}\times\mathcal{W}$ denote the individual state space. Next, let λ be a probability measure on $(\mathcal{S}, \mathcal{B}_s)$, where $\mathcal{B}_s$ is the Borel $\sigma-$algebra. For every Borel set $B \in \mathcal{B}_s$, let $\lambda(B)$ indicate the mass of agents whose individual state vectors lie in B. Finally, define a transition function $P : \mathcal{S}\times \mathcal{B}_s \rightarrow [0, 1]$ so that $P(x, B)$ defines the probability that a household with state x will have an individual state vector lying in B next period.

Definition (Stationary Equilibrium): A stationary equilibrium is a collection of value functions $v(x)$, a household policy $\{c(x), s(x), d'(x), m'(x), h'(x)\}$, probability measure, λ, and price vector (q, ρ) such that:

1. $c(x), s(x), d'(x), m'(x)$, and $h'(x)$ are optimal decision rules to the households' decision problem, given prices q and ρ.

2. Markets clear:

   (a) Housing market clearing: $\int_{\mathcal{S}} h'(x)d\lambda = H$, where H is fixed;

   (b) Rental market clearing: $\int_{\mathcal{S}}(h'(x) - s(x))d\lambda = 0$;

   where $\mathcal{S}$=$\mathcal{D} \times \mathcal{M} \times \mathcal{H} \times \mathcal{W}$.

3. λ is a stationary probability measure: $\lambda(B) = \int_{\mathcal{S}} P(x, B)d\lambda$ for any Borel set $B \in \mathcal{B}_s$.

2. Appendix B: Frictionless Analytical Results

Consider a problem of a homeowner who consumes all housing services yielded by the owned property (e.g., $s = h'$) but also chooses how much to invest into a rental property, h'_r.

For simplicity, we assume that mortgage interest payments are fully tax deductible ($\tau^m = 1$), and that there are no borrowing constraints, buying and selling costs, income uncertainty, or landlord utility penalty. The homeowner thus chooses (c, h', h'_r, m', d') to optimally solve:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c, h')$$

subject to initial conditions and

$$\begin{aligned} & c + d' - m' + qh' + qh'_r \\ & \leq w \;\; + (1+r)d - (1+r^m)\, m + \rho h'_r + qh + qh_r - \tau^y \widetilde{y} - \tau^h qh' - \tau^h qh'_r - \delta_0 qh' - \delta_r qh'_r, \end{aligned}$$

where

$$\widetilde{y} = w + rd + \rho h'_r - \left[r^m m + \tau^h qh' + \tau^h qh'_r + \delta_r qh'_r + \tau^{LL} qh'_r \right].$$

The corresponding first order conditions are:

$$\begin{aligned} c &: \beta u_c(c, h') - \lambda = 0, \\ h' &: \beta u_h(c, h') + \lambda(-\tau^y \frac{\partial \widetilde{y}}{\partial h'} - \tau^h q - \delta_0 q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'} = -\tau^y \tau^h q, \\ h'_r &: \lambda(\rho - \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} - \tau^h q - \delta_r q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} = \tau^y(\rho - \tau^h q - \delta_r q - \tau^{LL} q), \\ d' &: -\lambda + \lambda'(-\tau^y \frac{\partial \widetilde{y}'}{\partial d'} + (1+r)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}'}{\partial d'} = \tau^y r, \\ m' &: \lambda + \lambda^{'}(-\tau^y \frac{\partial \widetilde{y}'}{\partial m'} - (1+r^m)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}'}{\partial m'} = -\tau^y r^m. \end{aligned}$$

Combining the first order conditions with respect to c and h', we obtain the expression representing the user cost of a homeowner,

$$\frac{u_h(c_t, h')}{u_c(c_t, h')} = q(1 + (1 - \tau^y)\tau^h + \delta_0) - \frac{\lambda'}{\lambda} q'. \tag{1}$$

Similarly, the first order condition with respect to h_r' gives the asset pricing equation for a landlord in this frictionless economy:

$$\rho = \frac{q(1 + (1 - \tau^y)\tau^h + (1 - \tau^y)\delta_r - \tau^y \tau^{LL}) - \frac{\lambda'}{\lambda} q'}{(1 - \tau^y)}. \tag{2}$$

Equations 1 and 2 can be used to compare the cost of housing of a renter to that of a homeowner. Landlords can access deductions not available to homeowners, such as physical depreciation of the rental property and maintenance costs. However, rental property depreciates at a higher rate, and rental income (unlike user-occupied space) is taxable. Letting $C := 1 + (1 - \tau^y)\tau^h - \frac{\lambda'}{\lambda}$, then in steady-state, equations 1 and 2 become

$$\frac{u_h(.)}{u_c(.)} = q(C + \delta_0)$$

$$(1 - \tau^y)\rho = q(C + (1 - \tau^y)\delta_r - \tau^y \tau^{LL})$$

Clearly, the fact that imputed rental income from owner-occupied shelter is excluded from taxable income is of central importance when examining the decision of a homeowner to supply rental property. At our calibrated parameter values, $\frac{u_h(.)}{u_c(.)} < \rho$, primarily due to the tax treatment of rental income – a result consistent with **?**. Moreover, **?** show that when there is a spread between the return on deposits and the mortgage rate (as in here), then households do not simultaneously hold deposits and debt; see their Proposition 2. As a result, using the first order conditions with d' and m', the user cost and the landlord asset pricing equations above can be further simplified by substituting $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r}$ if the homeowner holds deposits, or $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r^m}$ if the homeowner holds a mortgage loan.

3. Appendix C: Solving the Model

Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 1.. In practice, the market clearing rent (ρ^*) and house price (q^*) are found by finding the (q^*, ρ^*) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of N agents are

$$\sum_{i=1}^{N} h_i'(q^*, \rho^*|x) = H \tag{3}$$

$$\sum_{i=1}^{N} s_i'(q^*, \rho^*|x) = H. \tag{4}$$

The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables (x). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties dramatically decreases the amount of time required to find the equilibrium relative to a more naive search algorithm.

3.1. The Algorithm

Let q_k represent the kth guess of the market clearing house price, let ρ_k represent a guess of the equilibrium rent, and let $\rho_k(q_k)$ represent the rent that clears the market for housing conditional on house price q_k. The algorithm that searches for equilibrium is based on the following excess demand functions

$$D_k^h(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k|x) - H \tag{5}$$

$$D_k^s(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k|x) - H. \tag{6}$$

The equilibrium prices q^* and ρ^* simultaneously clear the markets for housing and shelter, so

$$D_k^h(q^*, \rho^*) = 0 \tag{7}$$

$$D_k^s(q^*, \rho^*) = 0. \tag{8}$$

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price q_k.

2. Search for the rent $\rho_k(q_k)$ which clears the market for owned housing conditional on the current guess of the equilibrium house price, q_k. The problem is to find the value of $\rho_k(q_k)$ such that $D_k^h(q_k, \rho_k(q_k)) = 0$. This step of the algorithm requires re-solving the agents' optimization problem at each trial value of $\rho_k(q_k)$, simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function $D_k^h(q_k, \rho_k(q_k))$ is that conditional on q_k, it is a strictly decreasing function of ρ_k. Based on this property, $\rho_k(q_k)$ can be found efficiently using bisection.

3. Given that the *housing* market clears at prices $(q_k, \rho_k(q_k))$, check if this pair of prices also clears the market for *shelter* by evaluating $D_k^s(q_k, \rho_k(q_k))$.

   (a) If $D_k^s(q_k, \rho_k(q_k)) < 0$ and $k = 1$, the initial guess q_1 is too high, so set $q_{k+1} = q_k - \varepsilon$ and go to step (2). This initial house price guess q_1 is too high if $D_k^s(q_k, \rho_k(q_k)) < 0$ because $D_k^s(q_k, \rho_k(q_k))$ is decreasing in q_k.

   (b) If $D_k^s(q_k, \rho_k(q_k)) > 0$ set $k = k + 1$ and $q_{k+1} = q_k + \varepsilon$ and go to step (2).

   (c) If $D_k^s(q_k, \rho_k(q_k)) = 0$, the equilibrium prices are $q^* = q_k$, $\rho^* = \rho_k(q_k)$, so stop.

3.2. Solving for the Transition Path

This appendix describes the solution of the model along the perfect foresight transition path between two steady states. In the first time period, the economy is in the initial, high interest rate, high down payment steady state. In time period $t = 2$, the interest rate and minimum down payment unexpectedly, and permanently, decline. Let T represent the number of time periods that it takes for the economy to converge to the final steady state.[1] Let (q^*, ρ^*) and (q^{**}, ρ^{**}) represent the initial and final steady state equilibrium house price and rent. The transition path is a sequence of prices, $\{q_t, \rho_t\}_{t=1}^{T}$, along which the optimal decisions of households clear both the markets for shelter and housing. Solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady state problem described in the paper, because both current-period prices and future prices affect households' optimal decisions. Given a sequence of prices $\{q_t, \rho_t\}_{t=1}^{T}$, the dynamic programming problem is solved recursively, moving backwards in time from time period T.

The algorithm begins by setting the market clearing prices in periods $t = 1$ and $t = T$ equal to their initial and final steady state values, so $q_1 = q^*$, $\rho_1 = \rho^*, q_T = q^{**}$, $\rho_T = \rho^{**}$. Next, a guess is made for the remaining prices along the transition path, $\{q_t, \rho_t\}_{t=2}^{T-1}$. The transition path is found using the following algorithm:

1. Solve the household problem recursively, moving backward from period T, taking the sequence of prices $\{q_t, \rho_t\}_{t=1}^{T}$ as given.

2. Use the optimal decision rules to simulate data from the model for each period along the transition path.

3. Check for market clearing in each time period using the conditions listed in section 1.. If markets clear in all time periods, stop because the transition path has been found.

[1] In practice, we set $T = 30$, but find that the economy converges to the new steady state after 25 periods. The computed equilibrium is unchanged by extending the horizon to $T > 30$.

If markets do not clear, make a new guess of the transition path and go back to step 1.

4. Appendix D: SCF Data

The Survey of Consumer Finances (SCF) 1998 is used to construct the moments summarized in the paper. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of U.S. families. The total housing wealth is constructed as the total sum of all residential real estate owned by a household, and is taken to represent the housing wealth qh' in the model. Secured debt (i.e., debt secured by primary or other residence) is used as a model analog of the collateralized debt, m'. The model analogue of the total net worth (i.e., $d' + qh' - m'$) is constructed as the sum of household's deposits in the transaction accounts and the housing wealth (as defined above), net of the secured debt. The total household income reported in the SCF is taken to represent the total household income defined in the model as $y = w + rd' + I^{h'>s}[\rho(h' - s)]$. Both data and the SAS code are available at request, or can be found at the official Survey of Consumer Finances website.